Poojari's institute of science

Derivative

1. If u = $\frac{2bt}{1+t^2}$, $v = a[\frac{1-t^2}{1+t^2}]$ show that $\frac{du}{dv}=\frac{-b^2 v}{d^2 u}$. **[Mar.98]**
2. If y = $\sin^{-1}[\frac{5sinx+4cosx}{\sqrt{41}}]$, find $\frac{dy}{dx}$. **(Ans : 1)** **[Mar.96]**
3. Find the derivative of x sin x from the first principles w.r.t.x. **(Ans : xcos x+sin x)** **[Mar.06/Oct.96,2002]**
4. Find $\frac{dy}{dx}$, if y = $\cos^{-1}[\frac{1-x^2}{1+x^2}]$. **(Ans : $\frac{2}{1+x^2}$)** **[Oct.96]**
5. If y = sin-1 x, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx}=0$. **[Oct.96]**
6. Find $\frac{dy}{dx}$, if y = cot-1 $[\frac{3+4\tan x}{4-3\tan x}]$. **(Ans: -1)** **[Mar.97]**
7. Find $\frac{dy}{dx}$, if y = log $[e^{3x}(\frac{x+5}{x-2})^{7/2}]$ **(Ans: 3+$\frac{7}{2(x+5)}$ - $\frac{7}{2(x-2)}$)** **[Oct.97]**
8. If $x^y = e^{x-y}$, show that $\frac{dy}{dx} = \frac{\log x}{(1+log x)^2}$. **[Mar.84/Oct.89,97]**
9. If y = $x^x+(\sin x)^x$, find $\frac{dy}{dx}$. **(Ans. $X^x(1+\log x)+(\sin x)^x[x \cot x+ \log \sin x]$)** **[Mar.88/Oct.97]**
10. Find $\frac{dy}{dx}$, if y = $\tan^{-1}[\frac{\cos x}{1+\sin x}]$. **(Ans . $-\frac{1}{2}$)** **[Mar.98]**
11. Find $\frac{dy}{dx}$, if y = $(\sin x)^{\tan x} + (\cos x)^{\tan x}$. **(Ans. $(\sin x)^{\tan x}$ [1 +log sin x.$\sec^2$ x] + $(\cos x)^{\tan x}$ [-$\tan^2$x + log cos x.$\sec^2$ x])** **[Mar.98]**
12. If x = $at^2, y = 2at$, where t is a parameter, then show that xy $\frac{d^2y}{dx^2}$ + a =0. **[Mar.98]**
13. If y = $\cot^{-1}[\frac{1+15x^2}{2x}]$, find $\frac{dy}{dx}$. **(Ans. $\frac{5}{1+25x^2}$ - $\frac{3}{1+9x^2}$) .** **[Oct.98]**
14. If x + $\sqrt{xy}$ +y = 1, find $\frac{dy}{dx}$. **(Ans. $\frac{-\sqrt{y}(2\sqrt{x}+\sqrt{y})}{\sqrt{x}(2\sqrt{y}+\sqrt{x})}$)** **[Oct.98]**
15. By the first principle , find the derivative of log (2x + 30 w.r.t.x. **(Ans.$\frac{2}{(2x+3)}$)** **[Oct.98]**
16. If y = $\frac{1}{x+a}$, show that $y_n = \frac{(-1)^n.n!}{(x+a)^{n+1}}$. **[Mar.99]**
17. If y = log $[\frac{1-\cos x}{1+\cos x}]$, show that $\frac{dy}{dx}$= 2 cosec x. **[Mar.99]**
18. If y = 3 cos$\boldsymbol{\theta}$ – 2 $\cos^3\boldsymbol{\theta}$, x = 3sin$\boldsymbol{\theta}$ – 2 $\sin^3\boldsymbol{\theta}$, find $\frac{dy}{dx}$ at $\boldsymbol{\theta} = \frac{\pi}{3}$. **(Ans.$\sqrt{3}$)** **[Mar.99]**
19. If y = sin x , show that $\frac{d^ny}{dx^n}$ = sin $[x + \frac{n\pi}{2}]$. **[Oct.99]**
20. Find $\frac{dy}{dx}$, if y = $\sec^{-1}[\frac{1+x^2}{1-x^2}]$. **(Ans.$\frac{2}{1+x^2}$)** **[Oct.99]**
21. If sin(xy) = x cos y, find $\frac{dy}{dx}$. **(Ans. $\frac{\cos y - y\cos xy}{x\cos xy + x\sin y}$)** **[Oct.99]**
22. Find the derivative of f(x) = 7^{2x} by using first principles. **(Ans. 2(log7)7^{2x})** **[Mar.2000]**
23. If y = xe^{xy}, show that $\frac{dy}{dx} = \frac{y(1+xy)}{x(1-xy)}$. **[Oct.90/Mar.2000]**
24. Find $\frac{dy}{dx}$, if y = $\tan^{-1}[\frac{1+x\sin x}{x-\sin x}]$. **(Ans. $\frac{-1}{1+x^2}$ + $\frac{\cos x}{1+\sin^2 x}$)** **[Mar.2000]**
25. By using , first principle, find the derivative of $x\sqrt{x}$ w.r.t.x. **(Ans.$\frac{3}{2}\sqrt{x}$)** **[Oct.2000]**
26. If u=sin-1$[\frac{2x}{1+x^2}]$ and v=cos-1 $[\frac{1-x^2}{1+x^2}]$, show that $\frac{du}{dv} = 1$ **[Oct.2000]**

27. If y = (tan x)^x +4$^{\cos x}$, find $\frac{dy}{dx}$.
    (Ans.(tan x)^x (x sec x cosec x + log tan x)-4$^{\cos x}$. log 4 . sin x) **[Oct.2000]**
28. If x = a cos $\boldsymbol{\theta}$, y = b sin $\boldsymbol{\theta}$ show that $\boldsymbol{a}^2$[y $\frac{d^2y}{dx^2}$ + [$\frac{dy}{dx}$]^2]+b^2=0 **[Mar.2001]**
29. If y=tan-1 [$\frac{x}{\sqrt{1+x^2}-1}$], find $\frac{dy}{dx}$. (Ans. $\frac{-1}{1(1+x^2\,)}$] **[Mar.2001]**

Poojari's institute of science

Derivative

30. If x^p.y^q= (x + y)$^{p+q}$, show that $\frac{dy}{dx} = \frac{y}{x}$. **[Mar.80,93,2001]**
31. If y = e$^{m\cos^{-1}}$ x, show that (1=x^2) $\frac{d^2y}{dx^2} - x\,\frac{dy}{dx} - \;m^2y = 0$. **[Oct.78/2001]**
32. If x^{3}y^k =(x+y)$^{3+k}$, show that $\frac{dy}{dx}$=$\frac{y}{x}$. **[Oct.2001]**
33. If y=tan^{-1}[$\frac{\cos 4x+\sin 4x}{\cos 4x-\sin 4x}$], find $\frac{dy}{dx}$. . (Ans.4) **[Oct.2001]**
34. If tan-1 [$\frac{x^2-y^2}{x^2+y^2}$] = , show that $\frac{dy}{dx} = \frac{x(1-\tan a)}{y(1+\tan\alpha)}$. **[Mar.94,2002]**
35. If y=$(x)^{\sqrt{x}}$ + $(\sqrt{x}\,)^x$, find $\frac{dy}{dx}$.
    [Ans. $\frac{x^{\sqrt{x}}}{\sqrt{x}}$ [1+ $\frac{1}{2}\log x$] +$\frac{(\sqrt{x}\,)^x}{2}$ (1+log x)] **[Mar.2002/Oct.2002]**
36. Differentiate log(1+x^2)w.r.t. $\tan^{-1} x$. **(Ans. 2x)** **[Oct.2002]**
37. If $ax^2 + 2hxy + by^2$=0, show that $\frac{d^2y}{dx^2}$=0. **[Mar.03,06/Oct.04]**
38. If x = $\frac{a(\,1-t^2\,)}{(\,1+t^2\,)}$ and y = $\frac{2at}{(\,1+t^2\,)}$, show that $\frac{dy}{dx}$= $\frac{t^2-1}{2t}$ **[Mar.2003]**
39. If y=[$\frac{x^2}{x+1}$], find $\frac{dy}{dx}$ **(Ans. y[2(1+log x) - $\frac{x}{x+1}$ - log (x +1)])** **[Mar.2003]**
40. If y = α sec $\boldsymbol{\theta}$, x = α tan $\boldsymbol{\theta}$, show thaty^3 $\frac{d^2y}{dx^2}$=a^2. [Oct.2003]
41. Find $\frac{dy}{dx}$, if y = $\cot^{-1}$[$\frac{2-5x}{5+2x}$]. **(Ans. $\frac{1}{1+x^2}$)** [Oct.2003]
42. Find from the first principle, the derivative of f(x)=log α x, where α>0,α≠1, x∊ R$^+$ with respect to x. (Ans. $\frac{1}{x\log a}$) [Oct.87,2003]
43. Find from first principles the derivative of x cos x w.r.t x.
    (Ans. –x sin x+ cos x) [Mar.2004]
44. If y = cosec^{-1} [$\frac{1}{2x\sqrt{1-x^2}}$], find $\frac{dy}{dx}$. (Ans. $\frac{2}{\sqrt{1-x^2}}$) [Mar.2004]
45. Find $\frac{dy}{dx}$, if y = (cos x)$^{\log x}$ + (log x)^x
    (Ans. (cos x)$^{\log x}$[$\frac{\log\cos x}{x}$ + log x . tan x] + (log x)^x[$\frac{1}{\log x}$ + log (log x)]) **[Oct.2004]**
46. Find $\frac{dy}{dx}$, if y = (log x)^x+ x$^{\cos x}$
    (Ans.(log x)^x[$\frac{1}{\log x}$ + log (log x)] + x$^{\cos x}$[$\frac{\cos x}{x}$ - sin x log x]) **[Mar.2005]**
47. If y = x sin y, show that $\frac{dy}{dx} = \frac{y}{x(1-x\cos y)}$. **[Mar.2005]**
48. If y = αe^{2x} + be^{3x} , show that $\frac{d^2y}{dx^2}$ - 5 $\frac{dy}{dx}$ +6y =0. **[Mar.2005]**
49. If y = log$_5$ x + log$_x$5-5^x , find $\frac{dy}{dx}$. **(Ans. $\frac{1}{x\log 5} - \frac{\log 5}{x(\log x)^2}$ - 5^{x}log 5)** **[Oct.2005]**

50. If y = $(\sin^{-1}x)^2$, show that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} - 2 = 0$. **[Oct.2005]**

51. Differentiate $\cos^{-1}[\frac{1-x^2}{1+x^2}]$ w.r.t. $\tan^{-1}[\frac{2x}{1-x^2}]$ **(Ans. 1)** **[Oct.04/Mar.06]**

52. If $y=\tan^{-1}[\frac{5x+1}{3-x-6x^2}]$ show that $\frac{dy}{dx} = \frac{3}{1+(3x+2)^2} + \frac{2}{1+(2x-1)^2}$. **[Mar.08]**

53. If $2y=\sqrt{x+1} + \sqrt{x-1}$, show that $4(x^2-1)\frac{d^2y}{dx^2} + 4x\frac{dy}{dx} - y=0$ **[Mar.08]**

54. If $(x^2+y)^{17} = x^8y^{13}$, prove that $\frac{dy}{dx} = \frac{2y}{x}$. **[Oct.08]**